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Constant-Cutoff Approach to Pion–Nucleon Scattering

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Pion–nucleon scattering amplitudes as well as Born amplitudes are calculated in a constant-cutoff approach to the canonical quantization of skyrmions, with the subsidiary conditions imposed on the quantum fields and their conjugate momenta such that all infrared singularities from the zero-frequency modes are eliminated. It is shown that the Born terms with recoil corrections are reproduced by the pion–nucleon linear and quadratic interactions.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

where

$$U = \frac{2}{F_\pi} (\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1):

$$\mathcal{L}_{\text{Sk}} = \frac{1}{32e^2} \operatorname{Tr}[U^{+}\partial_{\mu}U, U^{+}\partial_{\nu}U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors [e.g., Adkins *et al.* (1983); for an extensive list of other references see Holzwarth and Schwesinger (1986) and Nyman and Riska (1990)] that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)], \qquad \hat{\mathbf{r}} = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated the possibility of building a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. They showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensionless parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^{+}(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4} F_{\pi}^2 \frac{1}{\chi''(0)} a + \frac{1}{2} \frac{[\chi''(0)]^3}{(\pi/4)F_{\pi}^2 b} J(J + 1) \tag{1.5}$$

where

$$a = \int_0^{\infty} \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4} \mathcal{F}\right)\right] dr \tag{1.6}$$

$$b = \int_0^{\infty} ds \frac{64}{3} s^2 \sin^2\left(\frac{1}{4} \mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5), with respect to the arbitrary dimensional scale parameters $\chi''(0)$, is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by MW is not correct since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy}\cos\left[\frac{\sqrt{7}}{2}\ln(cy) + \alpha\right]$$
$$m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen adequately. When $F(0) = -n\pi$ then we want to know which of these two solutions is approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\, dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\, dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b; 1992) I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4), as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to calculate the pion–nucleon scattering amplitudes as well as Born amplitudes, with the subsidiary conditions imposed on the quantum fields and their conjugate momenta such that all infrared singularities from the zero-frequency modes are eliminated (Ohta, 1990, 1991a,b). It is shown that the Born terms with recoil corrections are reproduced by the pion–nucleon linear and quadratic interactions.

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. Balakrishna *et al.* (1991) discussed an interesting analogy with the damped pendulum, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by equation (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been discussed in Dalarsson (1993, 1995a–c).

2. CONSTANT-CUTOFF STABILIZATION

The chiral soliton with baryon number $n = 1$ is given by (1.4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (1.4) into (1.1), we obtain the static energy of the chiral baryon

$$M = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced, following Balakrishna *et al.* (1991), as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U\, \partial^\mu U^+)\, d^3x \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t)U_0(\mathbf{r}, t)A^+(t), \qquad U^+(\mathbf{r}, t) = A(t)U_0^+(\mathbf{r}, t)A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3x = -M \tag{2.6}$$

is equal to minus the energy M given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$\begin{aligned} L &= \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3x \\ &= bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[\dot{x}(t)]^2 \end{aligned} \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2\! F\, y^2\, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2\, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$M = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2\left(\frac{dF}{dy}\right)^2 + 2\sin^2\! F\right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A\, \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$, $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion-decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. MATRIX ELEMENTS FOR PION–NUCLEON SCATTERING

3.1. Zero Modes and Field Fluctuations

The Lagrangian of the simplified Skyrme model with massive pions is obtained from (1.1) by adding the chiral-symmetry-breaking mass term, and is given by

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ + \frac{m_\pi^2 F_\pi^2}{16} \operatorname{Tr}(U + U^+ - 2) \tag{3.1}$$

Using (1.2), we may write

$$\mathcal{L} = \frac{1}{2} \dot{\pi}_i G_{ij} \dot{\pi}_j + \mathcal{M}(\pi) \tag{3.2}$$

where

$$\mathcal{M}(\pi) = \frac{1}{2} \partial_k \pi_i G_{ij} \partial_k \pi_j + \frac{1}{2} m_\pi^2 \pi_j^2, \qquad M = \int d^3\mathbf{r} \, \mathcal{M}(\pi) \tag{3.3}$$

and the metric G_{ij} is given by

$$G_{ij} = \delta_{ij} + \frac{\pi_i \pi_j}{\sigma^2} = \delta_{ij} + \tan^2 F \, \hat{r}_i \hat{r}_j \tag{3.4}$$

In the present paper, following Ohta (1990, 1991a,b), we split the pion isotriplet into its static part and the fluctuation

$$\pi_i(\mathbf{r}, t) = \phi_i(\mathbf{r}) + \psi_i(\mathbf{r}, t) \tag{3.5}$$

Expanding the Lagrangian up to second-order terms in the fluctuation $\psi_i(\mathbf{r}, t)$, we obtain the following differential equation for normal modes $\boldsymbol{\psi}(\mathbf{r}, t) = \boldsymbol{\psi}_n(\mathbf{r}) \exp(-i\omega_n t)$:

$$\mathcal{H}_{ij} \psi_{nj}(\mathbf{r}) = \omega_n^2 G_{ij} \psi_{nj}(\mathbf{r}) \tag{3.6}$$

where we use the box normalization and normal modes are labeled by their asymptotic momenta k_n, i.e., $\psi_n = \psi(k_n)$, and the operator $\mathcal{H}_{ij}$ is defined by

$$\mathcal{H}_{ij}(\boldsymbol{\phi}) = \frac{\delta^2 \mathcal{M}}{\delta\phi_i \delta\phi_j} + \frac{\delta^2 \mathcal{M}}{\delta(\partial_l \phi_i)\delta\phi_j} \partial_l - \partial_l \left[\frac{\delta^2 \mathcal{M}}{\delta\phi_i \delta(\partial_l \phi_j)} \right] - \partial_l \left[\frac{\delta^2 \mathcal{M}}{\delta(\partial_l \phi_i)\delta(\partial_m \phi_j)} \partial_m \right] \tag{3.7}$$

Operators G_{ij} and $\mathcal{H}_{ij}$ are now functions of the static part ϕ_i and its derivatives $\partial_m \phi_i$ only, such that $\mathcal{H}_{ij}$ can simultaneously be diagonalized with the operator of the combined spatial and isospin rotations $\mathbf{K} = \mathbf{J} + \mathbf{T}$ which leave the solution (1.4) invariant. Normal modes with definite K and K_3 are given by (Mattis and Karliner, 1985; Mattis and Peshkin, 1985; Karliner and Mattis, 1986)

$$\psi_{k_n}^{K,K_3,l}(\mathbf{r}) = R_{k_n}^{K,l}(r)\mathbf{Y}_{K,l}^{K_3}(\hat{\mathbf{r}}) \qquad (l = K - 1, K, K + 1) \tag{3.8}$$

where $R_n^{K,l}(r)$ is the radial part and $\mathbf{Y}_{K,l}^{K_3}(\hat{\mathbf{r}})$ are spherical vector harmonics. Normalization of the radial functions is defined by the asymptotic behavior and the pion plane wave is given by

$$\psi_{k_n}^{K,K_3,l}(\mathbf{r}) = \frac{4\pi i^l}{\sqrt{V}} j_l(k_n r)\mathbf{Y}_{K,l}^{K_3}(\hat{\mathbf{r}}) \tag{3.9}$$

In the following we will simplify notation by using k instead of k_n, and use a more convenient set of modes

$$\begin{bmatrix} \psi_k^{K,K_3(L)}(\mathbf{r}) \\ \psi_k^{K,K_3(M)}(\mathbf{r}) \\ \psi_k^{K,K_3(E)}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} \left(\dfrac{K}{2K+1}\right)^{1/2} & 0 & -\left(\dfrac{K+1}{2K+1}\right)^{1/2} \\ 0 & 1 & 0 \\ \left(\dfrac{K+1}{2K+1}\right)^{1/2} & 0 & \left(\dfrac{K}{2K+1}\right)^{1/2} \end{bmatrix} \begin{bmatrix} \psi_k^{K,K_3,K-1}(\mathbf{r}) \\ \psi_k^{K,K_3,K}(\mathbf{r}) \\ \psi_k^{K,K_3,K+1}(\mathbf{r}) \end{bmatrix} \tag{3.10}$$

where E, M, and L modes stand for electric, magnetic, and multipole longitudinal modes, respectively. Due to the translational and rotational invariances of the Lagrangian (3.1), pion waves with $K = 1$ contain zero-frequency modes. Properly normalized, these modes are given in the Cartesian representation by

$$\begin{aligned} \psi_0^{1,a(\mathrm{L})}(\mathbf{r}) &= M^{-1/2}\partial_a\phi_0 \\ &= \sqrt{\tfrac{1}{3}}\, R_0^{10}(r) \sum_m e_m^{a*}\mathbf{Y}_{10}^m(\hat{\mathbf{r}}) - \sqrt{\tfrac{2}{3}}\, R_0^{12}(r) \sum_m e_m^{a*}\mathbf{Y}_{12}^m(\hat{\mathbf{r}}) \end{aligned} \tag{3.11}$$

$$\psi_0^{1,a(\mathrm{M})}(\mathbf{r}) = \Omega^{-1/2} i T^a \phi_0 = R_0^{11}(r) \sum_m e_m^{a*}\mathbf{Y}_{11}^m(\hat{\mathbf{r}}) \tag{3.12}$$

where $iT_{ij}^a = \epsilon_{aij}$ is a matrix formed from the unit antisymmetric tensor ϵ_{aij} and Ω is the moment of inertia of the soliton, given by

$$\Omega\,\delta^{km} = \int d^3\mathbf{r}\, (iT^k\psi)^+ G i T^m \psi \tag{3.13}$$

The explicit form of the radial functions is

$$R_0^{10}(r) = \left(\frac{4\pi}{3M}\right)^{1/2} F_\pi\left(\cos F \frac{dF}{dr} + \frac{2 \sin F}{r}\right) \tag{3.14}$$

$$R_0^{12}(r) = \left(\frac{4\pi}{3M}\right)^{1/2} F_\pi\left(\cos F \frac{dF}{dr} - \frac{\sin F}{r}\right) \tag{3.15}$$

$$R_0^{11}(r) = i\left(\frac{4\pi}{3\Omega}\right)^{1/2} F_\pi \sin F \tag{3.16}$$

The explicit form of equation (3.6) is apparently dependent on the definition of the fluctuation, but following Holzwarth *et al.* (1990), we can remove this dependence by eliminating the metric G_{ij}, as follows:

$$\tilde{\mathcal{H}} = G^{-1/2}\mathcal{H}G^{-1/2} \tag{3.17}$$

$$\tilde{\boldsymbol{\psi}} = G^{1/2}\boldsymbol{\psi} \tag{3.18}$$

Equation (3.6) now becomes

$$\tilde{\mathcal{H}}\tilde{\psi}_n(\mathbf{r}) = \omega_n^2\tilde{\psi}_{nj}(\mathbf{r}) \tag{3.19}$$

and the modified zero modes are

$$\begin{aligned}\tilde{\boldsymbol{\psi}}_0^{1,a(\mathrm{L})}(\mathbf{r}) &= G^{1/2}\boldsymbol{\psi}_0^{1,a(\mathrm{L})} \\ &= \sqrt{\tfrac{1}{3}}\, \tilde{R}_0^{10}(r) \sum_m e_m^{a*}\mathbf{Y}_{10}^m(\hat{\mathbf{r}}) - \sqrt{\tfrac{2}{3}}\, \tilde{R}_0^{12}(r) \sum_m e_m^{a*}\mathbf{Y}_{12}^m(\hat{\mathbf{r}})\end{aligned} \tag{3.20}$$

$$\tilde{\boldsymbol{\psi}}_0^{1,a(\mathrm{M})}(\mathbf{r}) = G^{1/2}\boldsymbol{\psi}_0^{1,a(\mathrm{M})} = \tilde{R}_0^{11}(r) \sum_m e_m^{a*}\mathbf{Y}_{11}^m(\hat{\mathbf{r}}) \tag{3.21}$$

with the modified radial functions

$$\tilde{R}_0^{10}(r) = \left(\frac{4\pi}{3M}\right)^{1/2} F_\pi\left(\frac{dF}{dr} + \frac{2 \sin F}{r}\right) \tag{3.22}$$

$$\tilde{R}_0^{12}(r) = \left(\frac{4\pi}{3M}\right)^{1/2} F_\pi\left(\frac{dF}{dr} - \frac{\sin F}{r}\right) \tag{3.23}$$

$$\tilde{R}_0^{11}(r) = i\left(\frac{4\pi}{3\Omega}\right)^{1/2} F_\pi \sin F = R_0^{11}(r) \tag{3.24}$$

From (3.14), (3.15), (3.22), and (3.23) we see that in the asymptotic region ($r \to \infty$), where $\cos F \to 1$, the modified zero modes approach the original zero modes.

3.2. Canonical Transformation and the Hamiltonian

In order to perform the canonical coordinate quantization we now introduce the collective coordinates similar to those of (2.10), which we denote by $\boldsymbol{\rho}(t)$, $\boldsymbol{\alpha}(t)$, and $\boldsymbol{\chi}(t)$. The $\boldsymbol{\rho}(t)$ are the barycentric coordinates of the pion–nucleon system and $\boldsymbol{\alpha}(t)$ are the rotational coordinates (R) like those in (2.10). The canonical transformation of the field variables is now given by

$$\phi_i(\mathbf{r}, t) = R_{ij}[\alpha(t)]\{\phi_j[\mathbf{r} - \boldsymbol{\rho}(t)] + \chi_j[\mathbf{r} - \boldsymbol{\rho}(t), t]\} \tag{3.25}$$

and the canonical momentum conjugate to $\chi_a(t)$ is defined by

$$\Pi_a^\chi(\mathbf{r} - \boldsymbol{\rho}, t) = \frac{\delta\mathcal{L}}{\delta\dot{\chi}_a[\mathbf{r} - \boldsymbol{\rho}, t]} = \Pi_b^\phi(\mathbf{r}, t)R_{ba}(\boldsymbol{\alpha}) \tag{3.26}$$

where

$$\Pi_b^\phi(\mathbf{r}, t) = \frac{\delta\mathcal{L}}{\delta\dot{\phi}_b(\mathbf{r}, t)} = \Pi_b^\phi(\mathbf{r}, t)R_{ba}(\boldsymbol{\alpha}) \tag{3.27}$$

As argued in Gervais *et al.* (1976), there are six primary constraints between the collective coordinates and momenta

$$F_a^{\mathrm{T}} = P_a + \int d^3\mathbf{r}\, \Pi_b^\chi\, \partial_a(\phi_b + \chi_b) \approx 0 \tag{3.28}$$

$$F_a^{\mathrm{R}} = I_a + \int d^3\mathbf{r}\, \Pi_b^\chi iT_{abc}(\phi_c + \chi_c) \approx 0 \tag{3.29}$$

where P_a is the momentum conjugate to $\rho_a(t)$ and I_a is the body-fixed isospin. The momentum conjugate to $\alpha_a(t)$ is defined by $\not{p}_a = \partial\mathcal{L}/\partial\dot{\alpha}_a$. Introducing the modified fields

$$\tilde{\chi}_a(\mathbf{r}, t) = G_{ab}^{1/2}(\mathbf{r})\chi_b(\mathbf{r}, t) \tag{3.30}$$

$$\tilde{\Pi}_a^\chi(\mathbf{r}, t) = \Pi_b^\chi(\mathbf{r}, t)G_{ba}^{-1/2}(\mathbf{r}) \tag{3.31}$$

we find that the constraints (3.28) and (3.29) become

$$F_a^{\mathrm{T}} = P_a + \int d^3\mathbf{r}\, \tilde{\Pi}_b^\chi\, \tilde{\partial}_a(\phi_b + \chi_b) \approx 0 \tag{3.32}$$

$$F_a^{\mathrm{R}} = I_a + \int d^3\mathbf{r}\, \tilde{\Pi}_b^\chi i\tilde{T}_{abc}(\phi_c + \chi_c) \approx 0 \tag{3.33}$$

where $\tilde{\partial}_a\chi = G^{1/2}\partial_a\chi$ and $i\tilde{T}_a\chi = G^{1/2}iT_a\chi$. For quantization we now use the commutation relations

$$[\rho_a, P_b] = i\delta_{ab}, \qquad [\alpha_a, \not{p}_b] = i\delta_{ab} \tag{3.34}$$

In (3.28) and (3.29) as well as in (3.32) and (3.33) by wavy equality we indicate the weak condition since, although F_a^{T} and F_a^{R} vanish, their commutators with other operators do not necessarily vanish. In order to make these weak equalities strong, we use the Dirac formalism for quantization of constrained systems. Since we have added six extra collective coordinates, we have to impose six gauge-fixing conditions to preserve the number of quantum degrees of freedom. Generalizing the conditions considered in Gervais *et al.* (1976) from the two-dimensional to the three-dimensional case, we impose the subsidiary constraints

$$Q_a^{\mathrm{T}} = \int d^3\mathbf{r}\ \tilde{\boldsymbol{\chi}}\cdot\mathbf{f}_a^{\mathrm{T}} \approx 0 \tag{3.35}$$

$$Q_a^{\mathrm{R}} = \int d^3\mathbf{r}\ \tilde{\boldsymbol{\chi}}\cdot\mathbf{f}_a^{\mathrm{R}} \approx 0 \tag{3.36}$$

where $\mathbf{f}_a^{\mathrm{T}}$ and $\mathbf{f}_a^{\mathrm{R}}$ are some arbitrary real gauge-fixing functions. With these functions we also define two new parameters μ and θ as follows:

$$\mu\delta_{ab} = \int d^3\mathbf{r}\ \mathbf{f}_a^{\mathrm{T}}\cdot\mathbf{f}_b^{\mathrm{T}} \tag{3.37}$$

$$\theta\delta_{ab} = \int d^3\mathbf{r}\ \mathbf{f}_a^{\mathrm{R}}\cdot\mathbf{f}_b^{\mathrm{R}} \tag{3.38}$$

Furthermore, we note that the nonlinear constraints (3.32) and (3.33) are not convenient for quantization. We therefore linearize these by changing variables from $\tilde{\boldsymbol{\Pi}}^{\mathrm{X}}$ to $\boldsymbol{\Pi}^{\mathrm{X}}$, such that they become

$$F_a^{\mathrm{T}} = \int d^3\mathbf{r}\ \boldsymbol{\Pi}^{\mathrm{X}}\cdot\mathbf{f}_a^{\mathrm{T}} \approx 0 \tag{3.39}$$

$$F_a^{\mathrm{R}} = \int d^3\mathbf{r}\ \boldsymbol{\Pi}^{\mathrm{X}}\cdot\mathbf{f}_a^{\mathrm{R}} \approx 0 \tag{3.40}$$

This is performed using the canonical transformation in the symmetrized form

$$\tilde{\boldsymbol{\Pi}}^{\mathrm{X}}(\mathbf{r}, t)$$
$$= \boldsymbol{\Pi}^{\mathrm{X}}(\mathbf{r}, t) - \frac{1}{2}\left\{[f_a^{\mathrm{T}}, f_b^{\mathrm{R}}]\begin{bmatrix}(M_{\mathrm{X}}^{-1})_{ac}^{\mathrm{TT}} & (M_{\mathrm{X}}^{-1})_{ad}^{\mathrm{TR}} \\ (M_{\mathrm{X}}^{-1})_{bc}^{\mathrm{RT}} & (M_{\mathrm{X}}^{-1})_{bd}^{\mathrm{RR}}\end{bmatrix}\begin{bmatrix}p_c^{\mathrm{S}} + \int d^3\mathbf{r}\ \boldsymbol{\Pi}^{\mathrm{X}}\cdot\tilde{\partial}_c\boldsymbol{\phi} \\ l_d^{\mathrm{S}} + \int d^3\mathbf{r}\ \boldsymbol{\Pi}^{\mathrm{X}}\cdot i\tilde{T}_d\boldsymbol{\phi}\end{bmatrix} + \text{h.c.}\right\} \tag{3.41}$$

where $p_a^S = p_a - p_a^\pi$ and $I_a^S = I_a - I_a^\pi$ are the soliton momentum and body-fixed isospin, respectively, and

$$p_a^\pi = -\int d^3\mathbf{r}\ \mathbf{\Pi}^\chi \cdot \tilde{\partial}_a \boldsymbol{\chi} \tag{3.42}$$

$$I_a^\pi = -\int d^3\mathbf{r}\ \mathbf{\Pi}^\chi \cdot i\tilde{T}_a \boldsymbol{\chi} \tag{3.43}$$

are pion momentum and body-fixed isospin, respectively. Furthermore, the matrix M_χ is defined by

$$M_\chi = \begin{bmatrix} \mu\delta_{ab} & 0 \\ 0 & \theta\delta_{ab} \end{bmatrix} + \begin{bmatrix} \int d^3\mathbf{r}\ \mathbf{f}_b^T \cdot \tilde{\partial}_a \boldsymbol{\chi} & \int d^3\mathbf{r}\ \mathbf{f}_b^R \cdot \tilde{\partial}_a \boldsymbol{\chi} \\ \int d^3\mathbf{r}\ \mathbf{f}_b^T \cdot i\tilde{T}_a \boldsymbol{\chi} & \int d^3\mathbf{r}\ \mathbf{f}_b^R \cdot i\tilde{T}_a \boldsymbol{\chi} \end{bmatrix} \tag{3.44}$$

In order to make the weak equalities (3.32) and (3.33) strong, we replace the usual equal-times commutation relation for χ-fields

$$[\tilde{\chi}_a(\mathbf{r}, t), \Pi_b^\chi(\mathbf{z}, t)] = i\delta_{ab}\delta(\mathbf{r} - \mathbf{z}) \tag{3.45}$$

by the following commutation relation:

$$\begin{aligned} &[\tilde{\chi}_a(\mathbf{r}, t), \Pi_b^\chi(\mathbf{z}, t)] \\ &\quad = i\delta_{ab}\delta(\mathbf{r} - \mathbf{z}) - \frac{i}{\mu} f_{ia}^T(\mathbf{r}) f_{ib}^T(\mathbf{z}) - \frac{i}{\theta} f_{ia}^R(\mathbf{r}) f_{ib}^R(\mathbf{z}) \end{aligned} \tag{3.46}$$

Thus, following Dirac's terminology, we turn second-class constraints (3.32) and (3.33) with (3.45) into first-class constraints with (3.46). It should be noted that the arbitrariness of the functions $\mathbf{f}_a^T$ and $\mathbf{f}_a^R$ is kept throughout the quantization procedure.

The Hamiltonian of the pion–nucleon system in terms of the fields χ and Π^χ can be written in the form

$$\begin{aligned} H &= \frac{1}{2}\int d^3\mathbf{r}\ \Pi_a^\phi(\mathbf{r}, t) G_{ab}^{-1}[\phi(\mathbf{r}, t)] \Pi_b^\phi(\mathbf{r}, t) + \int d^3\mathbf{r}\ \mathcal{M}[\phi(\mathbf{r}, t)] \\ &= \frac{1}{2}\int d^3\mathbf{r}\ \Pi_a^\chi(\mathbf{r} - \boldsymbol{\rho}, t) R_{ba} G_{bc}^{-1}[R\phi(\mathbf{r} - \boldsymbol{\rho})] R_{cd} \Pi_d^\chi(\mathbf{r} - \boldsymbol{\rho}, t) \\ &\quad + \int d^3\mathbf{r}\ \mathcal{M}\{R[\phi(\mathbf{r} - \boldsymbol{\rho}) + \chi(r - \rho, t)]\} \end{aligned} \tag{3.47}$$

Rewriting (3.47) in terms of the modified fields $\tilde{\chi}$ and Π^χ and keeping only terms up to the second order in the pion field and up to the order $N_C^{-1/2}$ in the N_C^{-1} expansion, we obtain (Verschelde, 1988, 1989)

$$H = H_0 + H_{\mathrm{I}} + H_{\mathrm{II}} \tag{3.48}$$

where H_0 is the free-field Hamiltonian

$$H_0 = H_{\mathrm{COLL}} + H_{\mathrm{INTR}} + H_{\mathrm{COUP}} \tag{3.49}$$

with

$$H_{\mathrm{COLL}} = M + \frac{\mu}{2M^2}\,\mathbf{p}^2 + \frac{\theta}{2\Omega^2}\,\mathbf{I}^2 \tag{3.50}$$

$$H_{\mathrm{INTR}} = \frac{\mu}{2M^2}\,(\mathbf{p}^\pi)^2 + \frac{\theta}{2\Omega^2}\,(\mathbf{I}^\pi)^2 + \frac{1}{2}\int d^3\mathbf{r}\;\mathbf{\Pi}^\chi\cdot\mathbf{\Pi}^\chi + \int d^3\mathbf{r}\;U(\tilde{\chi},\phi) \tag{3.51}$$

$$H_{\mathrm{COUP}} = -\frac{\mu}{M^2}\,\mathbf{P}\cdot\mathbf{p}^\pi - \frac{\theta}{\Omega^2}\,\mathbf{I}\cdot\mathbf{I}^\pi \tag{3.52}$$

and

$$U(\tilde{\chi},\phi) = \frac{1}{2}\left\{\tilde{\chi}_i G_{im}^{-1/2}\,\frac{\delta^2\mathcal{M}}{\delta\phi_m\delta\phi_n}\,G_{nj}^{-1/2}\tilde{\chi}_j + \tilde{\chi}_i G_{im}^{-1/2}\,\frac{\delta^2\mathcal{M}}{\delta\phi_m\delta(\partial_l\phi_n)}\,G_{nj}^{-1/2}\tilde{\partial}_l\chi_j + \tilde{\partial}_k\chi_i G_{im}^{-1/2}\,\frac{\delta^2\mathcal{M}}{\delta(\partial_k\phi_m)\delta(\partial_l\phi_n)}\,G_{nj}^{-1/2}\tilde{\partial}_l\chi_j\right\} \tag{3.53}$$

In (3.49) H_{COLL} is the Hamiltonian of the collective degrees of freedom, H_{INTR} is the Hamiltonian for the intrinsic pion field, and H_{COUP} is the Hamiltonian that couples the collective motion and the intrinsic pion field.

The eigenstates of H_0 are noninteracting baryon–pion states. In order to find these eigenstates we observe that the eigenstate of H_{COLL} is $|I\ I_3\ J_3\ \mathbf{P}\rangle$, where I and J $(=I)$ are total isospin and spin, respectively, and $\mathbf{P}$ is the total momentum. The eigenstate of H_{INTR} is then denoted by $|\mathbf{q}, I_3^\pi\rangle$, where $\mathbf{q}$ is the pion momentum and I_3^π is the third component of the pion body-fixed isospin. Since the collective variables commute with the intrinsic pion field, the 1-baryon–1-pion states are a linear combination of $|\mathbf{q}, I_3^\pi\rangle \otimes |I\ I_3\ J_3\ \mathbf{P}\rangle$ such that H_{COUP} is diagonalized, i.e.,

$$|I\ I_3\ \mathbf{P};\ J\ J_3\ \mathbf{q}\rangle = \sum_\nu (I\ J_3 - \nu\ 1\ \nu|J\ J_3)|\mathbf{q},\nu\rangle \otimes |I\ I_3\ J_3 - \nu\ \mathbf{P}\rangle \tag{3.54}$$

where $\mathbf{p} = \mathbf{P} - \mathbf{q}$ is the baryon momentum. Similarly 1-baryon–2-pion states are obtained as

$$|I\, I_3\, \mathbf{P};\, J\, \mathbf{q};\, J'\, J_3'\, \mathbf{q}'\rangle = \sum_\lambda (J\, J_3' - \lambda\; 1\; \lambda | J'\, J_3')|\mathbf{q}', \lambda\rangle \otimes |I\, I_3\, P;\, J_3' - \lambda\, \mathbf{q}\rangle \tag{3.55}$$

The term H_I is of the order $\mathcal{O}(N_\mathrm{C}^{-1/2})$ and it describes the linear pion–nucleon interaction

$$H_\mathrm{I} = H_\mathrm{I}^\mathrm{T} + H_\mathrm{I}^\mathrm{R} = \frac{\mu}{2M^2}\left\{p_{a'}^\mathrm{S}\int d^3\mathbf{r}\,\mathbf{\Pi}^\mathrm{X}\cdot\tilde{\partial}_a\boldsymbol{\phi}\right\} + \frac{\theta}{2\Omega^2}\left\{l_{a'}^\mathrm{S}\int d^3\mathbf{r}\,\mathbf{\Pi}^\mathrm{X}\cdot i\tilde{T}_a\boldsymbol{\phi}\right\} \tag{3.56}$$

The term H_II is of the order $\mathcal{O}(N_\mathrm{C}^0)$ and it describes the quadratic pion–nucleon interaction

$$H_\mathrm{II} = H_\mathrm{II}^\mathrm{T} + H_\mathrm{II}^\mathrm{R} = \frac{\mu}{2M^2}\left(\int d^3\mathbf{r}\,\mathbf{\Pi}^\mathrm{X}\cdot\tilde{\partial}_a\boldsymbol{\phi}\right)^2 + \frac{\theta}{2\Omega^2}\left(\int d^3\mathbf{r}\,\mathbf{\Pi}^\mathrm{X}\cdot i\tilde{T}_a\boldsymbol{\phi}\right)^2 \tag{3.57}$$

3.3. Matrix Elements and Vertex Renormalizations

Normal mode solutions for the fields $\tilde{\chi}$ and Π^X are given by

$$\tilde{\boldsymbol{\chi}}(\mathbf{r}, t) = \sum_{n,a}{}' \frac{1}{(2\omega_n)^{1/2}}[\tilde{\boldsymbol{\chi}}_{n,a}(\mathbf{r})e^{-i\omega_n t}a_{n,a} + \text{h.c.}] \tag{3.58}$$

$$\mathbf{\Pi}^\mathrm{X}(\mathbf{r}, t) = \sum_{n,a}{}' \frac{-i\omega_n}{(2\omega_n)^{1/2}}[\tilde{\boldsymbol{\chi}}_{n,a}(\mathbf{r})e^{-i\omega_n t}a_{n,a} - \text{h.c.}] \tag{3.59}$$

The prime on the sums (3.58) and (3.59) indicates that the zero-frequency solutions

$$|\tilde{\boldsymbol{\chi}}_0^{1a(\mathrm{L})}\rangle = \frac{1}{\sqrt{M}}|\mathbf{f}_a^\mathrm{T}\rangle, \qquad |\tilde{\boldsymbol{\chi}}_0^{1a(\mathrm{M})}\rangle = \frac{1}{\sqrt{\Omega}}|\mathbf{f}_a^\mathrm{R}\rangle \tag{3.60}$$

are omitted in the sums. The normal modes $\tilde{\chi}_n$ therefore have the completeness property

$$\sum_n{}' |\tilde{\boldsymbol{\chi}}_n\rangle\langle\tilde{\boldsymbol{\chi}}_n| = 1 - \frac{1}{M}|\mathbf{f}_a^\mathrm{T}\rangle\langle\mathbf{f}_a^\mathrm{T}| - \frac{1}{\Omega}|\mathbf{f}_a^\mathrm{R}\rangle\langle\mathbf{f}_a^\mathrm{R}| \tag{3.61}$$

which leads to the following useful sum rules:

$$\sum_n{}' \, |\langle \tilde{\partial}_a \boldsymbol{\phi} \, | \, \tilde{\boldsymbol{\chi}}_n \rangle|^2 = M\left(1 - \frac{M}{\mu}\right) \tag{3.62}$$

$$\sum_n{}' \, |\langle i\tilde{T}_a \boldsymbol{\phi} \, | \, \tilde{\boldsymbol{\chi}}_n \rangle|^2 = \Omega\left(1 - \frac{\Omega}{\theta}\right) \tag{3.63}$$

The completeness property (3.61) also ensures the usual commutation relation between the a-operators, i.e.,

$$[a_{n,a}, a^+_{n',a'}] = \delta_{n,n'}\delta_{a,a'} \tag{3.64}$$

$$[a_{n,a}, a_{n',a'}] = [a^+_{n,a}, a^+_{n',a'}] = 0 \tag{3.65}$$

We now calculate the matrix element of the linear interaction $H_{\rm I}$ between the 1-baryon and 1-baryon–1-pion states. Using (3.54) and (3.59), we obtain the matrix elements of $H_I^{\rm T}$ and $H_I^{\rm R}$ in the form

$$\begin{aligned}
&\langle I\, I_3\, J_3\, \mathbf{p} \,|\, H_I^{\rm T} \,|\, I'\, I_3'\, p' + q';\, J'\, J_3'\, \mathbf{q}' \rangle \\
&\quad = \delta_{I,I'}\delta_{I_3,I'_3}\delta_{\mathbf{p},\mathbf{p}'+\mathbf{q}'}(I\, J_3\, 1\, J_3' - J_3 \,|\, J'\, J_3')Y_{1,J'_3-J_3}(\hat{\mathbf{q}}') \\
&\quad\quad \times \frac{\mu}{2M^2}\frac{-i\omega_{q'}}{(2\omega_{q'})^{1/2}}[\mathbf{p}^2 - (\mathbf{p}')^2]\frac{1}{q'}\langle \tilde{\partial}_a \boldsymbol{\phi} \,|\, \tilde{\chi}_q^{1a(\rm L)} \rangle
\end{aligned} \tag{3.66}$$

$$\begin{aligned}
&\langle I\, I_3\, J_3\, \mathbf{p} \,|\, H_I^{\rm R} \,|\, I'\, I_3'\, p' + q';\, J'\, J_3'\, \mathbf{q}' \rangle \\
&\quad = \delta_{I,I'}\delta_{I_3,I'_3}\delta_{\mathbf{p},\mathbf{p}'+\mathbf{q}'}(I\, J_3\, 1\, J_3' - J_3 \,|\, J'\, J_3')Y_{1,J'_3-J_3}(\hat{\mathbf{q}}') \\
&\quad\quad \times \frac{\theta}{2\Omega^2}\frac{-i\omega_{q'}}{(2\omega_{q'})^{1/2}}[I(I+1) - J'(J'+1)]\frac{1}{\sqrt{2}}\langle i\tilde{T}_a \boldsymbol{\phi} \,|\, \tilde{\chi}_q^{1a(\rm M)} \rangle
\end{aligned} \tag{3.67}$$

where it should be noted that there is no sum over a. The matrix elements of $H_{\rm II}^{\rm T}$ and $H_{\rm II}^{\rm R}$ between 1-baryon–1-pion states are obtained in the form

$$\begin{aligned}
&\langle I\, I_3\, \mathbf{p} + \mathbf{q};\, J\, J_3\, \mathbf{q} \,|\, H_{\rm II}^{\rm T} \,|\, I'\, I_3'\, p' + q';\, J'\, J_3'\, \mathbf{q}' \rangle \\
&\quad = \delta_{I,I'}\delta_{I_3,I'_3}\delta_{\mathbf{p}+\mathbf{q},\mathbf{p}'+\mathbf{q}'}\frac{\mu}{2M^2}\frac{i\omega_q}{(2\omega_q)^{1/2}}\frac{-i\omega_{q'}}{(2\omega_{q'})^{1/2}} \\
&\quad\quad \times \sum_\nu (I\, \nu\, 1\, J_3 - \nu \,|\, J\, J_3)(I\, \nu\, 1\, J_3' - \nu \,|\, J'\, J_3')\langle \tilde{\chi}_q^{1a(\rm L)} \,|\, \tilde{\partial}_a \boldsymbol{\phi} \rangle \\
&\quad\quad \times \langle \tilde{\partial}_a \boldsymbol{\phi} \,|\, \tilde{\chi}_q^{1a(\rm L)} \rangle \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}' Y^*_{1,J_3-\nu}(\hat{\mathbf{q}}) Y_{1,J'_3-\nu}(\hat{\mathbf{q}}')
\end{aligned} \tag{3.68}$$

$$\langle I\, I_3\, \mathbf{p} + \mathbf{q};\, J\, J_3\, \mathbf{q} | H_{\mathrm{II}}^{\mathrm{R}} | I'\, I_3'\, p' + q';\, J'\, J_3'\, \mathbf{q}' \rangle$$

$$= \delta_{I,I'}\delta_{I_3,I'_3}\delta_{\mathbf{p}+\mathbf{q},\mathbf{p}'+\mathbf{q}'} \frac{\theta}{2\Omega^2} \frac{i\omega_q}{(2\omega_q)^{1/2}} \frac{-i\omega_{q'}}{(2\omega_{q'})^{1/2}}$$

$$\times \sum_{\nu} (I\, \nu\, 1\, J_3 - \nu | J\, J_3)(I\, \nu\, 1\, J_3' - \nu | J'\, J_3')\langle \tilde{\chi}_q^{1a(\mathrm{M})} | i\tilde{T}_a \boldsymbol{\phi} \rangle$$

$$\times \langle i\tilde{T}_a \boldsymbol{\phi} | \tilde{\chi}_{q'}^{1a(\mathrm{M})} \rangle \frac{3}{8\pi} [\delta_{J_3,J'_3} \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}' - e_{J'_3-\nu}\cdot\hat{\mathbf{q}} e^*_{J_3-\nu}\cdot\hat{\mathbf{q}}'] \tag{3.69}$$

Now we calculate the vertex renormalizations of the Hamiltonians H_{II}, H_{I}, and H_0, respectively. The contribution of the Nth-order term in the vertex renormalization procedure to H_{II}, i.e., $\delta H_{\mathrm{II}}^{(N)}$, is obtained as, using sum rules (3.62) and (3.63),

$$\delta H_{\mathrm{II}}^{(N)} = \left(1 - \frac{\mu}{M}\right)^{N-1} H_{\mathrm{II}}^{\mathrm{T}} + \left(1 - \frac{\theta}{\Omega}\right)^{N-1} H_{\mathrm{II}}^{\mathrm{R}} \tag{3.70}$$

and the renormalized Hamiltonian is given by

$$H_{\mathrm{II}}^{(\mathrm{r})} = H_{\mathrm{II}} + \sum_{N=2}^{\infty} \delta H_{\mathrm{II}}^{(N)} = \frac{M}{\mu} H_{\mathrm{II}}^{\mathrm{T}} + \frac{\Omega}{\theta} H_{\mathrm{II}}^{\mathrm{R}}$$

$$= \frac{1}{2M}\left(\int d^3\mathbf{r}\, \boldsymbol{\Pi}^{\mathrm{X}}\cdot\tilde{\partial}_a\boldsymbol{\phi}\right)^2 + \frac{1}{2\Omega}\left(\int d^3\mathbf{r}\, \boldsymbol{\Pi}^{\mathrm{X}}\cdot i\tilde{T}_a\boldsymbol{\phi}\right)^2 \tag{3.71}$$

Similarly we obtain

$$\delta H_{\mathrm{I}} = \left(-1 + \frac{M}{\mu}\right) H_{\mathrm{I}}^{\mathrm{T}} + \left(-1 + \frac{\Omega}{\theta}\right) H_{\mathrm{I}}^{\mathrm{R}} \tag{3.72}$$

and the renormalized Hamiltonian becomes

$$H_{\mathrm{I}}^{(\mathrm{r})} = H_{\mathrm{I}} + \delta H_{\mathrm{I}} = \frac{M}{\mu} H_{\mathrm{I}}^{\mathrm{T}} + \frac{\Omega}{\theta} H_{\mathrm{I}}^{\mathrm{R}}$$

$$= \frac{1}{2M}\left\{p_a^{\mathrm{S}}, \int d^3\mathbf{r}\, \boldsymbol{\Pi}^{\mathrm{X}}\cdot\tilde{\partial}_a\boldsymbol{\phi}\right\} + \frac{1}{2\Omega}\left\{I_a^{\mathrm{S}}, \int d^3\mathbf{r}\, \boldsymbol{\Pi}^{\mathrm{X}}\cdot i\tilde{T}_a\boldsymbol{\phi}\right\} \tag{3.73}$$

In a similar manner, we obtain

$$\delta H_0 = \left(-1 + \frac{M}{\mu}\right) \frac{\mu}{2M^2} (\mathbf{P} - \mathbf{p}^{\pi})^2 + \left(-1 + \frac{\Omega}{\theta}\right) \frac{\theta}{2\Omega^2} (\mathbf{I} - \mathbf{I}^{\pi})^2 \tag{3.74}$$

and the renormalized Hamiltonian becomes

$$H_0^{(\mathrm{r})} = H_0 + \delta H_0 = H_{\mathrm{COLL}}^{(\mathrm{r})} + H_{\mathrm{INTR}}^{(\mathrm{r})} + H_{\mathrm{COUP}}^{(\mathrm{r})} \tag{3.75}$$

with

$$H^{(r)}_{\rm COLL} = M + \frac{1}{2M}\,\mathbf{p}^2 + \frac{1}{2\Omega}\,\mathbf{I}^2 \tag{3.76}$$

$$H^{(r)}_{\rm INTR} = \frac{1}{2M}\,(\mathbf{p}^\pi)^2 + \frac{1}{2\Omega}\,(\mathbf{I}^\pi)^2 + \frac{1}{2}\int d^3\mathbf{r}\;\mathbf{\Pi}^\chi\cdot\mathbf{\Pi}^\chi + \int d^3\mathbf{r}\;U(\tilde{\chi},\phi) \tag{3.77}$$

$$H^{(r)}_{\rm COUP} = -\frac{1}{M}\,\mathbf{P}\cdot\mathbf{p}^\pi - \frac{1}{\Omega}\,\mathbf{I}\cdot\mathbf{I}^\pi \tag{3.78}$$

Thus we see that the explicit dependence of the Hamiltonian on the parameters μ and θ has been removed as well as that the translational and rotational invariances are restored in the 1-pion–1-nucleon sector, independent of the choice of the gauge.

We now calculate the matrix element of the pion-field source-function $J_i(\mathbf{r}, 0) = \mathcal{H}_{0ij}\pi_j(\mathbf{r}, 0)$ between one-baryon states to obtain

$$\begin{aligned}&\langle \tfrac{1}{2}\, I_3'\, J_3'\, \mathbf{p}' \,|\, J_i(\mathbf{r}, 0) \,|\, \tfrac{1}{2}\, I_3\, J_3\, \mathbf{p}\rangle \\ &\qquad \simeq e^{-i\mathbf{q}\cdot\mathbf{r}} \langle \tfrac{1}{2}\, I_3'\, J_3' \,|\, \tau_i \boldsymbol{\sigma}\cdot\mathbf{q} \,|\, \tfrac{1}{2}\, I_3\, J_3\rangle i\, \frac{g_{\pi NN}(q^2)}{2M_N}\end{aligned} \tag{3.79}$$

where $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ and the πNN-vertex form factor is given by (Cohen, 1986)

$$\frac{g_{\pi NN}(q^2)}{2M_N} = \frac{2\pi}{3}\,\frac{\omega_q^2}{q}\,F_\pi \int_\epsilon^\infty r^2\,dr\, j_1(qr)\sin F \tag{3.80}$$

A similar procedure gives the $\pi N\Delta$-vertex form factor in the form

$$g_{\pi N\Delta}(q^2) = \frac{3}{\sqrt{2}}\, g_{\pi NN}(q^2) \tag{3.81}$$

4. GAUGE FIXING AND THE SCATTERING AMPLITUDES

In the present section we fix the gauge and determine the gauge functions $f_a^{\rm T}$ and $f_a^{\rm R}$ in such a way that the infrared singularities are removed from the pion wave functions and that they approach the plane wave solutions in the limit $\omega_k \to 0$, i.e., $\tilde{\chi}_{k,a} \to \phi_{k,a}$. By expanding $f_a^{\rm T}$ and $f_a^{\rm R}$ in terms of the $K = 1$ pion wave functions

$$\mathbf{f}_a^{\rm T} = \sum_k \gamma_k^{\rm T} \tilde{\psi}_k^{1a({\rm L})}, \qquad \mathbf{f}_a^{\rm R} = \sum_k \gamma_k^{\rm R} \tilde{\psi}_k^{1a({\rm M})} \tag{4.1}$$

we introduce two real gauge parameters $\gamma_k^{\rm T}$ and $\gamma_k^{\rm R}$. In order to satisfy the assumed normalization conditions

$$\int d^3\mathbf{r}\, \mathbf{f}_a^{\mathrm{T}} \cdot \tilde{\partial}_b \boldsymbol{\phi} = M\delta_{ab}, \qquad \int d^3\mathbf{r}\, \mathbf{f}_a^{\mathrm{R}} \cdot i\tilde{T}_b \boldsymbol{\phi} = \Omega\delta_{ab} \tag{4.2}$$

we must choose $\gamma_0^{\mathrm{T}} = \sqrt{M}$ and $\gamma_0^{\mathrm{R}} = \sqrt{\Omega}$. The examination of the infrared singularities arising from the zero-mode contribution in the $K = 1$ modes gives the asymptotic expression

$$|\tilde{\chi}_k^{1a(\mathrm{L})}\rangle \simeq |\phi_k^{1a(\mathrm{L})}\rangle - \left\{\langle\tilde{\psi}_0^{1a(\mathrm{L})}|\phi_k^{1a(\mathrm{L})}\rangle + \frac{1}{\sqrt{M}}\gamma_k^{\mathrm{T}}\right\}|\tilde{\psi}_0^{1a(\mathrm{L})}\rangle \tag{4.3}$$

including all infrared divergencies. From (4.3) it is clear that we can eliminate infrared divergencies if we choose γ_k^{T} as

$$\gamma_k^{\mathrm{T}} = -\sqrt{M}\,\langle\tilde{\psi}_0^{1a(\mathrm{L})}|\phi_k^{1a(\mathrm{L})}\rangle \tag{4.4}$$

Similarly we find that we must choose γ_k^{R} as follows:

$$\gamma_k^{\mathrm{R}} = -\sqrt{\Omega}\,\langle\tilde{\psi}_0^{1a(\mathrm{M})}|\phi_k^{1a(\mathrm{M})}\rangle \tag{4.5}$$

Recalling now (3.21) and (3.24), we obtain

$$\langle\tilde{\psi}_0^{1a(\mathrm{M})}|\phi_k^{1a(\mathrm{M})}\rangle = \frac{4\pi^{3/2}}{(3V)^{1/2}}\frac{\sqrt{2}}{\sqrt{\Omega}}F_\pi \int r^2\, dr\, j_1(kr)\sin F \tag{4.6}$$

Similarly we find

$$\langle\tilde{\psi}_0^{1a(\mathrm{L})}|\phi_k^{1a(\mathrm{L})}\rangle = \frac{4\pi^{3/2}}{(3V)^{1/2}}\frac{k}{\sqrt{M}}F_\pi \int r^2\, dr\, j_1(kr)\sin F \tag{4.7}$$

and due to (3.80), we obtain

$$\gamma_k^{\mathrm{T}} = -\frac{2(3\pi)^{1/2}}{\sqrt{V}}\frac{k^2}{\omega_k^2}\frac{g_{\pi NN}(k^2)}{2M_N} \tag{4.8}$$

$$\gamma_k^{\mathrm{R}} = -\frac{2(6\pi)^{1/2}}{\sqrt{V}}\frac{k}{\omega_k^2}\frac{g_{\pi NN}(k^2)}{2M_N} \tag{4.9}$$

Using (4.8) and (4.9) together with (3.66) and (3.67), we can now calculate the renormalized matrix element

$$\begin{aligned} &\langle I\, I_3\, J_3\, \mathbf{p}|H_1^{(\mathrm{r})}|I'\, I_3'\, p' + q';\, J'\, J_3'\, \mathbf{q}'\rangle \\ &= \delta_{I,I'}\delta_{I_3,I'_3}\delta_{\mathbf{p},\mathbf{p}'+\mathbf{q}'}(I\, J_3\, 1\, J_3' - J_3|J'\, J_3')Y_{1,J'_3-J_3}(\hat{\mathbf{q}}') \\ &\times \{E_{\mathbf{p},I} - E_{\mathbf{p}',I'}\}\,\frac{-i(6\pi)^{1/2}}{\sqrt{V}}\frac{q'}{\omega_{q'}^{3/2}}\frac{g_{\pi NN}(q'^2)}{2M_N} \end{aligned} \tag{4.10}$$

where

$$E_{\mathbf{p},I} = M + \frac{1}{2M}\mathbf{P}^2 + \frac{1}{2\Omega} I(I+1) \tag{4.11}$$

The matrix element (4.10) agrees with that obtained in Verschelde (1988, 1989) by entirely different means. We now calculate elastic pion–nucleon scattering amplitudes at the tree level using $H_1^{(r)}$ and $H_{II}^{(r)}$. In order to calculate the contribution of $H_1^{(r)}$ we observe that the second-order T-matrix consists of direct and crossed terms which are given by

$$T^{(2)}_{\mathrm{DIR},1/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 3\boldsymbol{\sigma}\cdot\mathbf{q}'\ \boldsymbol{\sigma}\cdot\mathbf{q}\ \frac{1}{\omega_q^2}\frac{(E_p - E_{p+q})^2}{\omega_q + E_p - E_{p+q}} \tag{4.12}$$

$$T^{(2)}_{\mathrm{DIR},3/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \frac{9}{2}\mathbf{S}^{+}\cdot\mathbf{q}'\ \mathbf{S}\cdot\mathbf{q}\ \frac{1}{\omega_q^2}\frac{(E_p - E^{\Delta}_{p+q})^2}{\omega_q + E_p - E^{\Delta}_{p+q}} \tag{4.13}$$

$$\begin{aligned} T^{(2)}_{\mathrm{CROSS},1/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \Bigg[& -\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}\frac{(E_p - E_{p-q'})^2}{-\omega_q + E_p - E_{p-q'}} \\ & + 6\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}\frac{(E_p - E^{\Delta}_{p-q'})^2}{-\omega_q + E_p - E^{\Delta}_{p-q'}}\Bigg] \end{aligned} \tag{4.14}$$

$$\begin{aligned} T^{(2)}_{\mathrm{CROSS},1/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \Bigg[& 2\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}\frac{(E_p - E_{p-q'})^2}{-\omega_q + E_p - E_{p-q'}} \\ & + \frac{3}{2}\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}\frac{(E_p - E^{\Delta}_{p-q'})^2}{-\omega_q + E_p - E^{\Delta}_{p-q'}}\Bigg] \end{aligned} \tag{4.15}$$

where E_p and E^{Δ}_p stand for $E_{p,1/2}$ and $E_{p,3/2}$, respectively, and we used $|\mathbf{q}| = |\mathbf{q}'|$ to refer to the limit $\omega_q = \omega_{q'} = 0$. The pion–nucleon vertex function is replaced by the physical coupling constant $g_{\pi NN}(-m_\pi)$. Next we turn to the contribution of $H_{II}^{(r)}$ and calculate the matrix elements (3.68) and (3.69) and obtain the first-order T-matrix elements rearranged in such a way as to display the same structure as $T_1^{(2)}$ obtained above:

$$\begin{aligned} T^{(1)}_{1/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \Bigg[& -3\boldsymbol{\sigma}\cdot\mathbf{q}'\ \boldsymbol{\sigma}\cdot\mathbf{q}\ \frac{1}{\omega_q^2}(E_p - E_{p+q}) \\ & + \boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}(E_p - E_{p-q'}) \\ & - 6\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}(E_p - E^{\Delta}_{p-q'})\Bigg] \end{aligned} \tag{4.16}$$

$$T^{(1)}_{3/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \left[-2\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}(E_p - E_{p-q'}) \right.$$
$$- \frac{3}{2}\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'\ \frac{1}{\omega_q^2}(E_p - E^{\Delta}_{p-q'})$$
$$\left. - \frac{9}{2}\mathbf{S}^{+}\cdot\mathbf{q}'\ \mathbf{S}\cdot\mathbf{q}\ \frac{1}{\omega_q^2}(E_p - E^{\Delta}_{p+q}) \right] \tag{4.17}$$

where we have used $|\mathbf{p}| = |\mathbf{p}'|$ following from $|\mathbf{q}| = |\mathbf{q}'|$ and energy conservation. The T-matrix elements (4.16) and (4.17) agree with the zero-mode part of the background amplitude, i.e., the $\mathcal{O}(N_C^0)$ part of the Born terms (Kawarabayashi and Ohta, 1989). This is what we might expect since the $\mathcal{O}(N_C^0)$ term H_{II} appears due to the present choice of the canonical transformation, which eliminates the zero-mode term in the background scattering. Summing up all the terms above, we obtain

$$T_{1/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \left[3\,\frac{\boldsymbol{\sigma}\cdot\mathbf{q}'\ \boldsymbol{\sigma}\cdot\mathbf{q}}{\omega_q + E_p - E_{p+q}} - \frac{\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'}{-\omega_q + E_p - E_{p-q'}} \right.$$
$$\left. + 6\,\frac{\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'}{-\omega_q + E_p - E^{\Delta}_{p-q'}} \right] \tag{4.18}$$

$$T_{3/2}(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) = \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \left[2 - \frac{\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'}{-\omega_q + E_p - E_{p-q'}} \right.$$
$$+ \frac{3}{2}\,\frac{\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'}{-\omega_q + E_p - E^{\Delta}_{p-q'}}$$
$$\left. + \frac{9}{2}\,\frac{\mathbf{S}^{+}\cdot\mathbf{q}'\ \mathbf{S}\cdot\mathbf{q}}{\omega_q + E_p - E^{\Delta}_{p+q}} \right] \tag{4.19}$$

Substituting now $g_{\pi N\Delta}$ defined by (3.81) and rewriting the isospin projection operators $\hat{P}_I$ in terms of Pauli matrices and isospin transition operators, we can rearrange the scattering amplitude

$$T = \sum_I \hat{P}_I T_I \tag{4.20}$$

in the familiar form of the Born terms

$$\langle j | T(\mathbf{p}', \mathbf{q}'; \mathbf{p}, \mathbf{q}) | i \rangle$$
$$= \left[\frac{g_{\pi NN}}{2M_N}\right]^2 \left[\frac{\boldsymbol{\sigma}\cdot\mathbf{q}'\ \boldsymbol{\sigma}\cdot\mathbf{q}\ \tau_j\tau_i}{\omega_q + E_p - E_{p+q}} + \frac{\boldsymbol{\sigma}\cdot\mathbf{q}\ \boldsymbol{\sigma}\cdot\mathbf{q}'\ \tau_i\tau_j}{-\omega_q + E_p - E_{p-q'}} \right]$$
$$+ \left[\frac{g_{\pi N\Delta}}{2M_N}\right]^2 \left[\frac{\mathbf{S}^{+}\cdot\mathbf{q}'\ \mathbf{S}\cdot\mathbf{q}\ T_j^{+}T_i}{\omega_q + E_p - E^{\Delta}_{p+q}} + \frac{\mathbf{S}^{+}\cdot\mathbf{q}\ \mathbf{S}\cdot\mathbf{q}'\ T_i^{+}T_j}{-\omega_q + E_p - E^{\Delta}_{p-q'}} \right] \tag{4.21}$$

The results obtained so far agree with those obtained using the complete Skyrme model or conventional methods. The specific feature of the constant-cutoff approach is the implicit dependence of the scattering amplitudes on the constant cutoff ϵ through $g_{\pi NN}(\epsilon) = g\epsilon^3$, $g_{\pi N\Delta}(\epsilon) = (3/\sqrt{2})g\epsilon^3$, $M = a\epsilon$, and $\Omega = b\epsilon^3$. The constant cutoff ϵ itself is obtained by optimizing $H^{(r)}_{\mathrm{COLL}}$. Thus, neglecting the pion-mass-term contribution proportional to $m_\pi^2\epsilon^2$, being typically of the order 10^{-2} of the total soliton mass in the low-energy region of interest, we obtain the constant cutoff

$$\epsilon = \left[\frac{1}{4a^2}\mathbf{P}^2 + \left(\frac{1}{16a^4}\mathbf{P}^4 + \frac{3}{2ab}\mathbf{I}\right)^{1/2}\right]^{1/2} \tag{4.22}$$

In the limit $\mathbf{P} \to 0$, we see that it agrees with the result (2.12). Using (4.22) in (4.11), we obtain

$$E_{\mathbf{p},I} = a\epsilon + \frac{1}{2a\epsilon}\mathbf{P}^2 + \frac{1}{b\epsilon^3}I(I+1) \tag{4.23}$$

where $a = 0.78$ GeV^2 and $b = 0.91$ GeV^2 for $F_\pi = 186$ MeV, and the integral g above is obtained from (3.80) in the obvious way.

5. CONCLUSIONS

We have shown how to use the Skyrme model for the calculation of the pion–nucleon interaction matrix elements and scattering amplitudes without the use of the Skyrme stabilizing term, which is proportional to e^{-2}, which makes the practical calculations more complicated and requires some low-energy approximations which otherwise are not needed to obtain the correct Born terms for the scattering amplitude.

For such a simple model with only one arbitrary dimensional constant F_π we have shown that a heuristic approach to the choice of the subsidiary gauge-fixing conditions provides us with a theory with all the divergences removed and the translational and rotational invariances (although apparently violated at first) preserved. In the present approach the quadratic interaction H_{II} turns out to be the dominant one, although it is zero in the conventional gauge. Furthermore, we reproduce the Born terms using tree diagrams calculated from the renormalized H_{I} and H_{II}. The use of the present form of the Hamiltonian is not limited to the tree diagrams and low energies and it is possible for any higher order diagrams.

REFERENCES

Adkins, G. S., Nappi, C. R., and Witten, E. (1983). *Nuclear Physics B*, **228**, 552.
Balakrishna, B. S., Sanyuk, V., Schechter, J., and Subbaraman, A. (1991). *Physical Review D*, **45**, 344.

Bhadhuri, R. K. (1988). *Models of the Nucleon.* Addison-Wesley, Reading, Massachusetts.
Cohen, T. D. (1986). *Physical Review D,* **34**, 2187.
Dalarsson, N. (1991a). *Modern Physics Letters A,* **6**, 2345.
Dalarsson, N. (1991b). *Nuclear Physics A,* **532**, 708.
Dalarsson, N. (1992). *Nuclear Physics A,* **536**, 573.
Dalarsson, N. (1993). *Nuclear Physics A,* **554**, 580.
Dalarsson, N. (1995a). *International Journal of Theoretical Physics,* **34**, 81.
Dalarsson, N. (1995b). *International Journal of Theoretical Physics,* **34**, 949.
Dalarsson, N. (1995c). *International Journal of Theoretical Physics,* **34**, 2129.
Gervais, J. L., Jevicki, A., and Sakita, B. (1976). *Physics Reports,* **23**, 237.
Holzwarth, G., and Schwesinger, B. (1986). *Reports on Progress in Physics,* **49**, 825.
Holzwarth, G., Pari, G., and Jennings, B. K. (1990). *Nuclear Physics A,* **515**, 665.
Iwasaki, M., and Ohyama, H. (1989). *Physical Review,* **40**, 3125.
Jain, P., Schechter, J., and Sorkin, R. (1989). *Physical Review D,* **39**, 998.
Karliner, M., and Mattis, M. P. (1986). *Physical Review D,* **34**, 523.
Kawarabayashi, K., and Ohta, K. (1989). *Physics Letters B,* **216**, 205.
Mattis, M. P., and Karliner, M. (1985). *Physical Review D,* **31**, 2833.
Mattis, M. P., and Peshkin, M. (1985). *Physical Review D,* **32**, 58.
Mignaco, J. A., and Wulck, S. (1989). *Physical Review Letters,* **62**, 1449.
Nyman, E. M., and Riska, D. O. (1990). *Reports on Progress in Physics,* **53**, 1137.
Ohta, K. (1990). *Physics Letters B,* **242**, 33.
Ohta, K. (1991a). *Physical Review D,* **43**, 2635.
Ohta, K. (1991b). *Nuclear Physics A,* **534**, 513.
Skyrme, T. H. R. (1961). *Proceedings of the Royal Society A,* **260**, 127.
Skyrme, T. H. R. (1962). *Nuclear Physics,* **31**, 556.
Verschelde, H. (1988). *Physics Letters B,* **215**, 444.
Verschelde, H. (1989). *Physics Letters B,* **232**, 15.